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How teachers can use TI-Nspire CAS with laptops in an upper secondary course

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Abstract

The focus of this study is how teachers can use laptops in their classes with TI-Nspire CAS technology and software, with or without concomitant use of handheld devices. Of particular interest has been examining possible changes in teachers' classroom practice and attitudes in using this technology for improving students' mathematical learning, problem-solving methods and deeper understanding of mathematics. Eight classes of students in theoretical programmes at upper secondary level in Sweden had continuous access to TI-Nspire CAS in mathematics during a whole semester. They used the software, and in some classes handhelds, during a whole course and also implemented the national test for the course on their laptops.

The results show how TI-Nspire software on laptops can be used in regular education in courses at upper secondary level. Its various possibilities, of technical, mathematical and conceptual nature, have had the opportunity to appear in this relatively long study. But also the various obstacles and risks of this type of technology were identified, and the teachers' approaches to them were recorded. The teachers, having quite different prior experiences of technology, showed significant progress during the study, both in terms of management of technology in mathematical work, and when it came to integrating it into a high-quality learning environment. They also testified of the positive impact that the use of technology had on their students' view of mathematics and of what mathematical activities would include.

1. Introduction

Calculators and computer software have been used for a rather long period in mathematics classrooms. A development of the calculators (handheld units) has taken place through the years, from basic calculators to graphing ones, and now advanced calculators working with computer algebra systems (CAS) and with dynamic graphs and geometry (DGS). During the same time, computers have changed from being large and rather rare in mathematics education into smaller, mobile units (laptops) that can more easily be used in instruction with continuity. The software has simultaneously changed from more particular mathematics programs to more general ones. One observation is that calculators and computer software show a converging development, even if there are differences in the practical use of them. They can be combined through a system of software and hand units that gives the user the opportunity to choose when and where he/she wants to use the one or the other. The TI-Nspire system, with or without CAS, can be used either as handheld units or as computer software, or as a combination of the two.

Much of published research of technology used in mathematics instruction has been limited to handheld calculators, also when CAS has been used. Thus, it is of great value to also study how teachers and students are able to use laptops with TI-Nspire technology as software, with or without the simultaneous use of handheld units, and with the constructed curriculum material as an optional aid. Of special interest is furthermore to investigate possible changes in teaching practice, of students' problem-solving methods and of students' mathematical learning and deeper understanding of mathematics, as well as other outcomes of this technological environment for teaching.

2. Theoretical framework

The theoretical background for this evaluation rests on the classical *didactic triangle* with its three main elements *student-teacher-mathematics*, discussed for example by Steinbring [12]. This model has, however, been presented in various ways, depending on the overarching theory of learning and on the special context. The focus here lies on processes of mathematical interaction between individuals in the classroom [4], a mainly social constructivist view. Learning takes place through experiences that are mediated by tools, that can be mental (like spoken language), symbolic (like mathematical signs) or physical (like compasses), and with assistance drawn from other, competent individuals. Calculators and computer software hold a special position here, as they can be seen as tools within all three aspects (Figure 1).

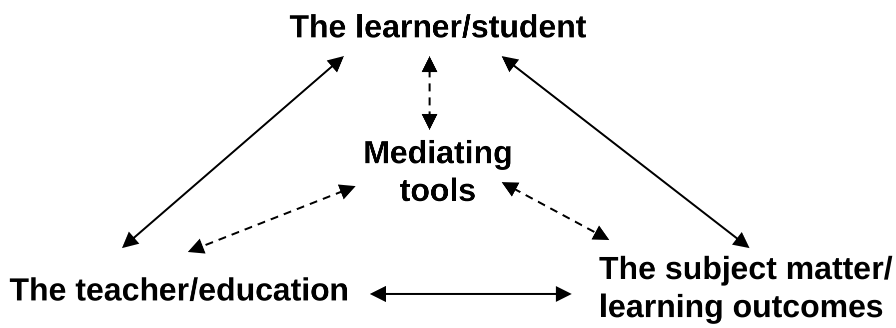


Figure 1: *the didactic triangle with mediating tools as facilitators*

Balling [3] distinguishes between the use of software and calculators as *calculating tools*, *teaching tools* and *learning tools*. When they are used mainly for facilitating calculations (extensions of the calculators used before), they function as calculating tools. When the teacher takes advantage of their possibilities to illustrate and show important features of concepts and methods, they are used as teaching tools. Finally, when students use them for exploring mathematical objects, to discover concept features and to solve problems, they have the role of learning tools.

A tool can develop into a useful *instrument* in a learning process called *instrumental genesis* [7], which has two closely interconnected components; *instrumentalization*, directed toward the artefact, and *instrumentation*, directed toward the subject, the student (See fig.2). These processes require time and effort from the user. He/she must develop skills for recognizing the tasks in which the instrument can be used and must then perform these tasks with the tool. For this, the user must develop *instrumented action schemes* that consist of a *technical part* and a *mental part* [7]. To learn instrumentation schemes does not in itself induce mathematical meaning and knowledge. Instead, the teacher must actively guide the students in a controlled evolution of knowledge, achieved by means of social construction in a class community [9]. Of special interest is the *instrumental orchestration*, which is defined as the intentional and systematic organisation and use of the artefacts available in a learning environment by the teacher, in order to guide students' instrumental genesis [5]. In the present research project, TI-Nspire CAS calculators together with the emulating computer software are the physical parts of the instrumentation process.

The term *resources* is used to emphasize the variety of artefacts we can consider: a textbook, a piece of software, a student's sheet, a discussion, etc. [6]. A resource is never isolated; it belongs to a set of resources. A process of genesis takes place, producing what is called a document. The teacher and the students build schemes of utilization of a set of resources for the same class of situations across a variety of contexts. This process is called a *documentational genesis* and also needs time and effort [6]. The participation and identity in the mathematical classroom builds on integrated communication and on representational infrastructures [8]. The way this is realised in teaching practice decides the effectiveness of information transfer and of cooperation, both student-student and teacher-student.

The TI-Nspire environment has been studied for example by Artigue and Bardini [2]. They give a list of why this type of technology can be considered as novel and special, such as its nature, its file organizing and navigation system, its dynamic connection between graphical and geometrical environments and lists/spread sheets as well as its possibilities to create variables that can be used in any of the pages and applications within an activity. In their results, they noted that:

> *…the introduction of this new tool was not without difficulty and required considerable initial work on the part of the teachers, both to allow rapid familiarisation on their part and those of the pupils but also to actualize the potentials offered by this new tool in mathematics activities* (p. 1179).

They also claim that:

> *These characteristics affect teachers and students differently, and individuals belonging to the same category differently, according to their personal characteristics and experience. They can have both positive and negative influences on teaching and learning processes and need to be better understood* (p. 1179).

Aldon [1] has studied the use of TI-Nspire calculators, and assumes that the calculator is both a tool allowing calculation and representation of mathematical objects but also an element of students' and teachers' sets of resources [6]. As a digital resource, these handheld calculators possess the main functions required for documentary production. Also Weigand and Bichler [13] have studied the use of calculators, and they formulate some interesting questions for research. These concern how to counter the polarization that seems to occur between different students in the use of technology, the relationship between uncertainty among students in technical handling of the unit and lack of knowledge regarding the use of it in a way that is appropriate for the particular problem, and about the problem that some students are taking a very long time to establish such a familiarity with the tool that they can use it in an adequate way.

3. Aims and research questions

The intention was to make a study of the use of TI-Nspire CAS technology, as software for laptops (Figure 2) and as software combined with handheld calculators, in some upper secondary classes, where each student has continuous access to his/her own laptop and can use it for mathematics as well as for communication over the net (intranet and Internet). Six classes with laptops participated, of which one had concomitant access to hand units. Two classes with only handheld units acted as control group, and 133 students in total from the theoretical programmes (Natural Science and Social Science) participated together with their teachers.

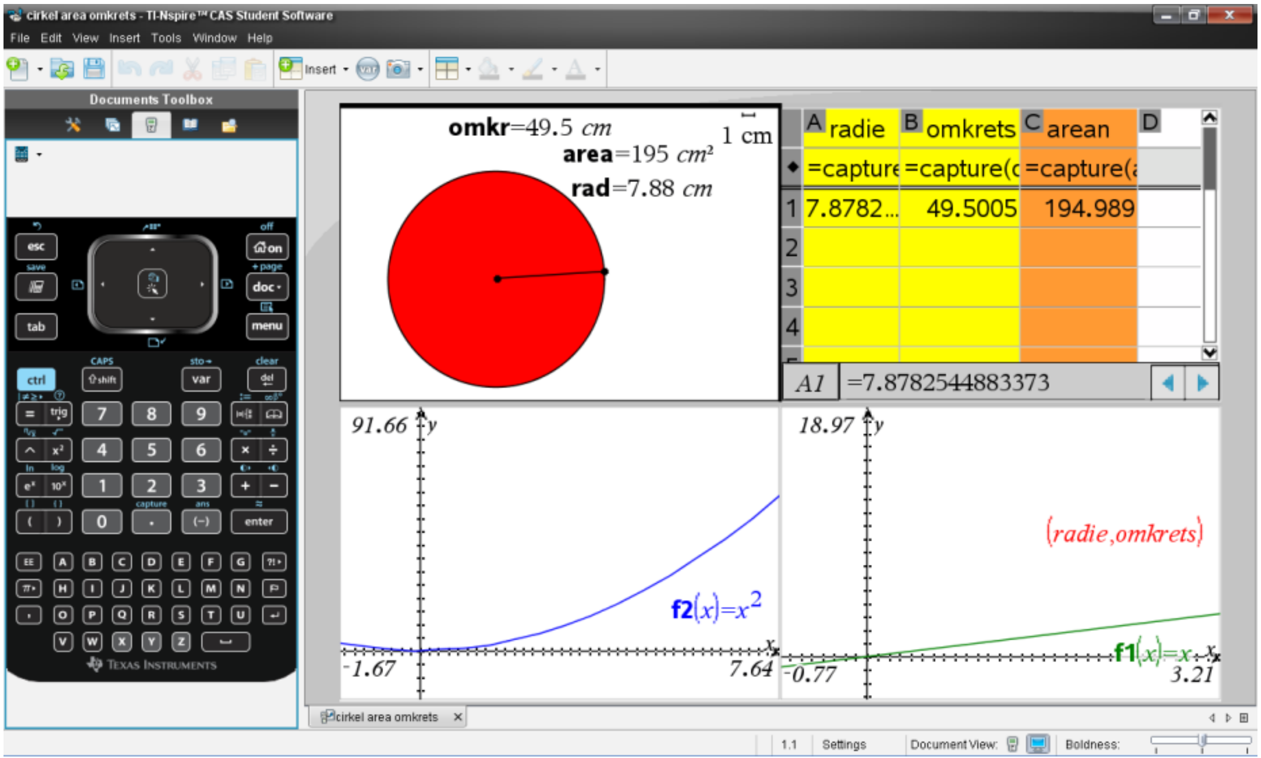


Figure 2: *the appearance of the TI-Nspire software, including the virtual keyboard of the calculator and split representations*

Of special interest for the study were possible changes in teachers' instructional practice and the effect this has on students' classroom work when they migrate from their current handheld (in most cases graphing calculators) to either version of TI-Nspire CAS or the combination of the two. This double shift, both from a graphing to a symbolic tool and from handheld to laptop, is not without complications [2]. There exists a need for further research of this shift. A special aim was to discern the advantages with using both handheld and laptops in the classroom work, and if important features and possibilities of the technology can be missing when only laptops are used.

Teachers, as well as students, had the opportunity to show and also to express their opinions of the use of this material and this technology, especially compared to other learning tools like ordinary textbooks and graphing calculators or software, e.g. Geogebra. However, one of the main questions is the effects of this special learning environment, of which TI-Nspire here is a vital part, on students' ability to solve problems and on their mathematical knowledge and conceptual understanding.

The research questions were structured according to the three corners of the 'didactical triangle'. In this paper, however, only the parts of the study that are mainly from a teaching perspective will be discussed:

1. Which benefits and special values, or obstacles, do teachers express of the two types of learning environment with TI-Nspire CAS, with laptops or with both laptops and handhelds, especially in comparison with other types of learning environments?
   In particular, does the use of handhelds together with laptops add extra values to the teaching opportunities?

2. Which examples of how the instrumental and the documentational geneses have progressed during the project can be found? In particular, are there differences between the environments with and without handhelds units?
3. How has these digital tools supported new approaches to teaching for the teachers involved in the research project, leading to a change in their teaching practice? What common obstacles to new methods of teaching using these tools have they detected?
4. Which examples can be found of how the teachers have used the possibilities of the technology intentionally to promote student reflection on mathematical methods and concepts?

The complete report of the entire study can be retrieved at Malmö University's database [10]. Some parts of the study have also been discussed in an article by the author[11]. These documents also address questions from the students' perspective.

4. Methodology

The classes and the teachers were all visited twice during the project. In comparing the data collected at the different occasions, it was possible to detect signs of progression in a variety of ways, such as teaching practice, the students' use of the material and the technology, dialog and collaborative learning in the classroom, conceptual understanding etc. The second visit also included a special problem-solving experiment aimed at detecting the students' skills and knowledge in using TI-Nspire CAS-technology for calculating, problem-solving and reflecting on answers and results that the technology presents for them.

The methods used involved the following main parts:

- *Teacher interviews*. A semi-structured interview with the teachers was made in connection with the first visit at the schools. All interviews were recorded and later transcribed.
- *Student interviews*. Two students were chosen from each class to be interviewed in semi-structured form directly after the observed lesson by the first visit. And directly after the teaching experiment a focus group of 5-6 students were interviewed about their experience of the task and of TI-Nspire in general. Both of these types of interviews were also recorded and transcribed.
- *Classroom observations*. At the first visit at each school a lesson was observed by the researcher, using a special observation form.
- *Teaching experiment*. In the later part of the course, all students participated in a problem-solving experiment, conducted by the researcher, and designed to detect the students' ability to use the TI-Nspire technology in a versatile way in longer, exploring task, and to record and communicate the result in a documental form (tns-file). A suitable problem-solving task was constructed within the area of linear functions and inequalities, which is an important part of the course curriculum.
- *Teacher questionnaire*. At the end of the school year all teachers were given questions concerning their overall experience of using the material and the different combinations of technology in their teaching practice, as well as their estimations of the effects on students' deeper understanding of mathematical concepts and methods and the effects on students' motivation, interest and self-confidence in connection with mathematics. The questionnaire was in its whole net-based.

- *Student questionnaire.* All participating students had the opportunity to express their experience of the learning environment with the types of technology they have used, their estimations of the quality of the mathematical learning with it, and of how it has affected their motivation, interest and self-confidence. Also this questionnaire was net-based.
- *Collection of material.* The intention was that interesting teaching material, tasks, tests, etc. that the participating teachers had produced during the project would be collected. Of special interest were the results of the national tests, providing an opportunity to detect possible differences between the classes using both handhelds and laptops and those who only had access to laptops. Some samples of the students’ produced tns-files and how they are organized in folders were also to be collected.

All data, of qualitative as well as of quantitative nature, was classified and structured according to the research questions, with the didactic triangle as the overarching structure. The method of analysis for the qualitative data is mainly descriptive [12]. Combined data from the interviews and the questionnaires give an interesting view of the advantages as well as the difficulties with using TI-Nspire technology.

5. Some findings

A considerable amount of data was collected during the study, and here only a selection of the results obtained will be shown. The focus in this article is the teachers' perspective, but in the tables below also students' opinions are given. The reason for this is to show whether or not teachers' and students' views coincide.

Comparing advantages and risks

The tables 1 – 6 below mainly addresses research question 1. They are the result of a compilation of the analysis of questionnaires and interviews.

Table 1: *common advantages with using TI-Nspire technology. Remarks about how frequent the answer is and some results from the questionnaires are given in the comments column [10]*

Advantages	Teachers	Students	Comments
A clear and distinct screen.	X	X	Frequent in interviews
Fast and flexible to work with.		X	Rather frequent
Easier to present new concepts and demonstrate in whole class.	X	X	5 teachers
Easy and useful for work with functions and graphs.	X	X	Frequent, 70 % in st.q. all teachers
New possibilities in the geometry and chance areas of mathematics.		X	
You can write all of the solutions to tasks in the program/on the handheld.		X	
You can easily check answers, also those you solve by hand.	X	X	

You can manage more difficult tasks,on a higher level.	X	X	Rather freq., 42 % in st.q. 6 teachers
New tools, like the solve-command, give you more power.		X	Rather frequent
You can learn more and understand mathematics better.	X	X	3 teachers
You can use several ways to solve a problem.	X	X	6 teachers
You can focus on understanding instead of making many calculations.	X		
Easy to use after a while.	X	X	
Useful in other subject, e.g. physics and chemistry.		X	
Easier to communicate.	X		3 teachers
Mathematics is more interesting with TI-Nspire		X	24 % in st.q.
More fun to work with mathematics	X	X	34 % in st.q. 6 teachers
You cooperate more in problem-solving		X	21 % in st.q.
The use of TI-Nspire has changes my conceptions of how you work with mathematics.		X	26 % in st.q.

Table 2: *common difficulties with using TI.Nspire technology. Remarks about how frequent the answer is and some results from the questionnaires are given in the comments column [10]*

Difficulties and risks	**Teachers**	**Students**	**Comments**
Hard to start with TI-Nspire.	X	X	6 teachers
Students think it is hard to use in ‘normal’ schoolwork.	X		6 teachers
Takes time to learn how to use TI-Nspire, e.g. find your way in menus.	X	X	Rather freq., 47 % in st.q. 3 teachers
Difficult to use different tools, e.g. for functions and graphs.		X	Rather freq., 39 % in st.q.
Sometimes difficult to know how to start solving a problem.		X	Rather freq., 42 % in st.q.
Sometimes you do not know what you are doing, especially using CAS.		X	
Sometimes hard to interpret the answers you get with CAS, e.g. with the solve-command.		X	26 % in st.q.
CAS difficult to handle. The step up from graphing calculators is high.	X		
It is essential that you also practice solving tasks with paper and pencil. You must do both.	X		Frequent in the interviews
When you work with paper and pencil, you understand better.	X		

A risk for the less able students that they cannot manage this technology, especially CAS.	X		
A risk for the less able students that they learn less than without technology.	X		Rather freq. in interviews
Technology often brings problems of technical nature, e.g. empty batteries, starting up etc.	X	X	Frequent in interviews

Table 3: *advantages with TI-Nspire compared to paper-and-pencil work. Remarks about how frequent the answer is and some results from the questionnaires are given in the comments column [10]*

Advantages with TI.Nspire compared to paper-and-pencil work	**Teachers**	**Students**	**Comments**
You work faster, so you reach further in mathematics and you get better knowledge.	X	X	Frequent by students
Nicer and more accurate graphs	X	X	Freq. by both t. and st.
You can make more difficult algebraic calculations.		X	
You can easily try many alternatives, e.g. for a function.		X	
You have usually many alternatives to how to solve a problem.	X	X	
You work more in groups than with p-o-p.		X	
You can focus more on understanding e.g. a graph and less on plotting and drawing it.	X	X	
Better understanding of mathematics with TI-Nspire.	X		
Easier to check answers, which is rarely done with p-o-p.	X	X	Frequent by students

Table 4: *advantages with laptops compared to handheld units. Remarks about how frequent the answer is and some results from the questionnaires are given in the comments column [10]*

Advantages with laptops compared to handheld units.	**Teachers**	**Students**	**Comments**
Larger screen with colour. You see more of what you are doing. User friendly.	X	X	Frequent in interviews.
Easier to work with a whole keyboard	X	X	
You can use the usual key commands for computers.		X	
Easier to edit expressions and text		X	
Easier to find your way in menus	X	X	
Better for handling tns-files	X	X	

Table 5: *advantages with handheld units compared to laptops. Remarks about how frequent the answer is and some results from the questionnaires are given in the comments column [10]*

Advantages with handheld units compared to laptops.	**Teachers**	**Students**	**Comments**
Faster with handheld when you are doing simpler calculations.		X	Rather freq. by students
Therefore more flexible in other subjects, e.g. physics.	X	X	
Easier with handhelds in test situations.	X	X	
Easier to carry than a laptop.		X	
Therefore less risk forgetting to bring.		X	
It takes more time to start the computers.	X		
More technical problems with computers.	X		
You are not dependent on a network.	X	X	

Table 6: *advantages with having handheld units combined with laptops. Remarks about how frequent the answer is and some results from the questionnaires are given in the comments column [10]*

Advantages with having handheld units combined with laptops.	**Teachers**	**Students**	**Comments**
You can choose yourself which is best in each situation if you are used to both.		X	
Handheld units are better to use at tests, but computers in the everyday work.	X		
Handheld units are better for quick calculations, computers for working with graph or solving larger problems	X	X	Rather freq. by students
Easier with transfer of files when you do it yourself.		X	
You are not so dependent on a network.	X	X	

Many of these are well-known opinions of teachers and students that have been presented in other research of the use of technology in general. But the difference is that this research project deals with the use of laptops in regular teaching over a longer period. Here are some teachers' voices (T1 etc. are abbreviations for teachers):

T1: I am very positive to using that type of tool. I think you get a much better understanding, an eye-opener, and not as much tinkering by hand with miscalculations. You get a much better picture, and it binds better ties between math and physics as well.

T2: I welcome it, because I think it can increase understanding. You can check calculations, make your own calculations and test different ways of calculating. One can see how mathematics can be related. Then I think it might be a little more fun and interesting, hopefully. That you do not always work exactly the same as with the book, but you can work in different ways. I hope the students may think it is fun to explore and learn, get some wow-experiences.

Critical opinions were also expressed by some:

T1: It's a pretty steep learning curve, I think. It's been 1-2 months now and only now they have really started to get acquainted with everything. In the beginning, it was quite chaotic.

T2: I fear that the students who has trouble keeping up with the others too easily use the calculator to see that it got right what he did, without really thinking through the task itself. I fear that they will enter 'solve' to see what happens. Then you do not get this struggling like you get when sitting with pencil and paper.

Advantages with laptops, hand units or the combination of the two:

T1: One advantage with technology is that it is faster. You can more quickly get to what is important in mathematics. If I have to draw something on the board, without technology, it takes a very long time, and then of course the students are asleep when I do it. Then it's really good with this, one can immediately draw and then you have mathematics.

Student: It is much more comfortable to sit with a calculator in a test instead of a computer in front of you. And the handheld is very pleasant to work with when you want to get something fast. It is also easier to move around and carry a calculator than a computer.

T2: User-friendliness is very much better on the computer software than on the calculator, so it is easier to use. And it's bigger and better with colour screen. And a little bit easier also with file handling. Users can post files that students can download. It's easier than if you were to send out files with "connect-to-class", with this as an extra task on the calculators.

T3: It gets much clearer on the computer with graphs. It has more space to explore in them. For students, despite having the computers, handhelds are many times better. So it's both. They use both continuously.

To make a summary of the opinions of the equipment that the teachers and students had used: Most of them where satisfied with what they had, and did not want to change. But the combination of laptop and handheld unit protrudes somewhat, in that almost all the students who had that equipment believed and gave reasons for this being the best, and this was confirmed by their teachers. They expressed that they wanted to have a choice in a given situation.

Using laptops at the national test

All six classes with laptops could, by special permission by the Swedish National Agency for Education, use them in the national test. There were two main conditions for this: First, any communication between students or through Internet was forbidden, and second, unwanted files that could be used for cheating should not be accessible. Only the software TI-Nspire was allowed for the students to use. Five of the teachers solved the problems with the two conditions by positioning themselves behind the students, so that they could watch all screens the whole time.

T1: Students sat in a large classroom all facing forward. I stood in the back of the classroom so that I could see all the computers. For questions they had to come to me, not I to them, because to the students should not know which way I looked. We cannot turn off the wireless network. This interferes with other activities too much.

One of the teachers instead applied the method of the closed down network:

T2: We used the computers in both tests. The school had shut down Internet access just for the computers that the students used when they made the tests. We had no action against bluetooth, but students did not use this, I am quite sure.
We had to place students with computer to computer and back to back with a larger wooden screen between the computers when they were sitting next to each other. Students appreciated having them in the test, since CAS is much clearer on the computer than on the hand units.

The teachers answered that there were no or very minor problems in the test situations, so the overall result of this point in the study is that it is possible to manage national tests with laptops, and also that there are various ways to fulfil the conditions. These results from the study were in fact part of the basis for a later decision of the Swedish National Agency to permit laptops at national tests. They are now allowed under the conditions mentioned above.

Instrumental and documentational genesis

In the beginning of the project, most of the teachers were rather unfamiliar with the TI-Nspire software and the handheld units. They were, as mentioned above, also rather new in using CAS in mathematics teaching. In the teacher interview they were asked about in what ways they used the laptops or the handhelds. The alternatives were: for demonstration during reviews, for general discussion in class, for helping students or groups of students. The answers in the teacher interviews varied quite a lot, mainly depending on what skills the individual teacher had, or believed he/she had. Here are some examples:

T: My reviews, of course, and then students can work simultaneously. And it is clear that when you move around in the class and help, you obviously take advantage of the software and show them and try to make them understand how to use it. Group discussions can of course also be very good sometimes, when they are sitting working and are forced to try to explain to each other.

The teachers were also asked in the interviews about how they intended the students to work with the technology. The alternatives here were: as a calculating aid, as a problem-solving tool, to discover and understand mathematical concepts and methods etc. [3]. Again, the answers in the interview varied:

T: Since I am a beginner, then it is the first option, of course. The second one, I am going with, but the third one... I have not got that far myself. But it's something I can imagine doing.

But most of the teachers also told that they wanted the students to help each other, and that it was very positive if they did so. The reasons for this are that it is good for the students to think and try for themselves before they get a sometimes too quick help, and that is important that students talk and discuss mathematics with each other. And some of the students often have acquired good skills in handling the technology.

T1: Usually it is the students who have the knowledge of the more practical management. There is always someone in the class who knows, and then knowledge is transmitted through students more often than through me. If anything pops up during the lesson they most often help each other out.

T2: It feels better when they are trying themselves for a while before they ask me, of course. If they ask me directly and I help them, not much has been solved for them actually. They have just been served a solution without having worked the problem. So I prefer that they do not call me directly. And I always encourage cooperation. To discuss the problems together and help each other I see as a great resource in the classroom.

The problem-solving experiment

With the problem-solving experiment near the end of the project, students' general skills in using the software or the handheld units were put to a test. The problems that the students were presented to were constructed with three levels: First involving ordinary calculations and/or readings of graphs, then some more complicated calculations with comparing different answers and making decisions, and last an exploring task where the students had to write answers in plain text. The intention was to give them the opportunity to show progress in the use of this technology in all three aspects of tools according to Balling's [3] classification.

One of the problems was called "Intersection points" and was based on two functions, one quadratic and one linear that intersected each other ($f_1(x) = x^2 + 1$ and $f_2(x) = 2x + 4$). First the students were asked to read and note the points of intersection, also in the case when the linear function was moved (by changing the constant) so that no intersection appeared. Then they had to find out with which constant term in the linear function (instead of 4) you get two, one or no intersections. After that they were asked to solve a non-linear system of equations that in reality exactly reflected the graphs in the first part (the students were supposed to discover this). Then a parameter *m* was introduced in the linear function for the constant term, and they were asked to solve the system again and explained why this general solution created two, one or no solutions for the system:

$\begin{cases} x^2 - y = -1 \\ 2x - y = -m \end{cases}$, with the solutions $\begin{cases} x = -\sqrt{m} + 1 \\ y = m - 2\sqrt{m} + 2 \end{cases}$ or $\begin{cases} x = \sqrt{m} + 1 \\ y = m + 2\sqrt{m} + 2 \end{cases}$

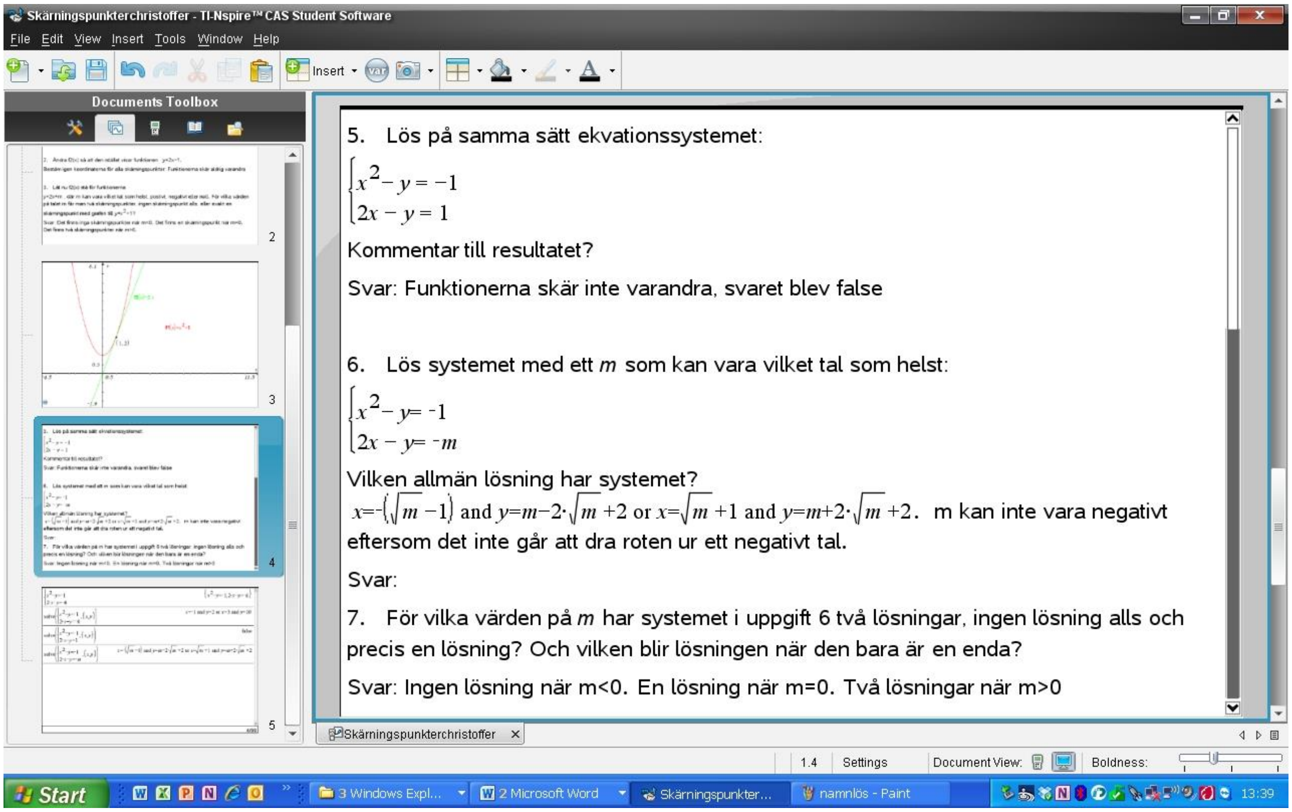
The students then were asked to reflect on the two general solutions and explain why these created the different types of solutions for varying values of *m*.

The observation of the classes and the analysis of their saved files showed that they handled TI-Nspire in a mainly productive way. Their problem-solving skills with TI-Nspire were good, with only a few exceptions. Many also managed to provide comprehensive answers to the more difficult parts of the problems (See figure 3 for one example). The experiences of the teachers were also in general that the students' development had progressed well throughout the project:

T1: They already invent their own methods to check things out. They have found the true-false function to check if expressions are equal. I have not taught them this. We see that they are a bit faster at detecting patterns, too.

T2: The more they learn about the technology the freer they become. And there's always a bunch which are doing the other way, with trial and error, and it does not work so well in the long run. But you must work really hard on it so that they get it as their tool.

Figure 3: *example of one student's answers to the last questions in the problem (in Swedish). To question 5 he writes: The functions do not intersect. The answer was "false". To question 6 he gives the general solution the CAS provides and writes: m cannot be negative, because it is impossible to take the square root of a negative number. In question 7 he concludes: No solution when m < 0. One solution when m = 0. Two solutions when m > 0*



6. Summary and discussion

This research project, with the different methods used, has created a lot of data. Some parts of the data point in somewhat different directions, but this is to be expected when you make research involving people. Humans are individuals, with different beliefs, interests and goals. These can create obstacles when new technology is introduced, especially in a special subject like mathematics, where beliefs about what counts as proper activities and methods go deep. And that is true for both teachers and students, as well as parents, headmasters and others in society, whose opinions do not show in this study.

Some interesting and important conclusions have been possible to draw, particularly of the benefits and difficulties of using laptops, with or without handheld units. These conclusions involve sometimes rather superficial things like appearance and similarity with computers, but also for mathematics education crucial things like the importance of problem solving and exploration and development of deeper understanding of mathematical concepts and methods. The results also shows that teachers in general are positive about the use of technology that TI-Nspire CAS, with its many opportunities and its mathematical robustness. A summary of some of the results [10]:

- The teachers expressed a number of *advantages* with the TI-Nspire technology in general. Among these were more *physical* benefits, as a good screen and that it is fast and flexible to work with. But more important were the mathematical ones, such as easier to work with functions and other areas of mathematics, new ways of working with problem-solving, dealing with difficult tasks and etc., and the conceptual ones, like learning more mathematics, understanding it better and focussing more on understanding in the activities.
- Among the *risks and difficulties* with the TI-Nspire that teachers mentioned were that it is difficult to begin with and that it takes some time for the students to learn to use. But after the initial "break-in period" the technology seemed to be pretty easy to use for them. Special difficulties were expressed in connection with the CAS, like it is difficult to handle and to understand the answers, and that sometimes the students do not really know what they do.
- Some benefits of the *TI-Nspire compared to paper and pencil* were also mentioned. The students work faster, so that they reach further into mathematics and get a better understanding. It is easier to work with graphs, to carry out difficult algebraic calculations and to try many options. To check the response data from the screen was also more frequent among students, and understanding of mathematics was promoted when they did not have to focus on simple calculations or plotting graphs.
- Benefits of *laptops compared to hand units* that were mentioned: larger screen with color so you can see more of what you do, easier to work with full keyboard, more user-friendly, easier to edit and navigate in menus, easier to handle files and more. Benefits of *hand units compared to laptops* were that they are easier to use in basic calculations and therefore more flexible in other disciplines such as physics, easier to handle in test situations, easier to carry than a laptop and therefore less likely to be forgotten and that they are not dependent on a network. It also takes more time to start the computers and that there are more technical problems in connection with these. Benefits of having *hand units combined with laptops* are that you can choose by yourself which of them that is best in each situation if you are used to both. Hand units are better for quick calculations, computers for working with graphs or to solve larger problems. It is also easier to transfer files when you have access to the entire system, and it is not so dependent on a network that might not work so well. Most of the teachers believed that a combination of handheld units and laptops is the ideal situation in the total classroom work.
- There was a high correlation between the benefits and special values of the three types of TI-Nspire environments that the teachers and the students mentioned. This is important for the decisions to start using this technology in mathematics at schools and in classrooms.
- At the *national tests* laptops were used without any larger problems. The method used by most of the teachers was to position them during the test so that it was possible for them to watch all the students' laptop screens. This particular experiment was successful, but also showed that more technical solutions are not chosen at first hand.
- Most of the teachers stated that their *ways of teaching* had changed to some extent. The general changes they stated were that they used computer and projector more, that they worked more with problem-solving and that they used group work more in their teaching. The ways they intended the students to work with the technology, as a calculating aid, as a

problem-solving tool, to discover and understand mathematical concepts and methods etc., varied to some extent but their explicit goal was all of these alternatives.

- Among common *obstacles to high-quality teaching* were that students could have difficulties in handling the technology and/or the mathematics they encounter there. Most of the teachers told that they wanted the students to help each other, because it is good for the students to think and try for themselves before they get a sometimes too quick help, and that it is important that students talk and discuss mathematics with each other.
- Few examples could be seen in the project of how the teachers had used the technology *intentionally to promote student reflection* on mathematical methods and concepts. For instance, most of the teachers did not construct their own tns-files for such a purpose. However, some explained that the cooperation between students is of great importance also for reflection, and reflection is important for the understanding of mathematics.
- The ways in which students *documented their work* with tasks and problems showed very little progress during the project. Most of them used paper and pencil to document, which also was what the majority of the teachers wanted them to do. But two of the classes, using laptops, were exceptions, in that they were used to the teacher giving them tasks as tns-files which they were to return with their solutions written in. A clear example of differences between environments with and without handheld units in documenting is that it is more difficult to work with text using the handhelds. The display is rather small and so is the keyboard. This makes it difficult to acquire any higher speed in the work with more complicated problems.
- All of the teachers answered that the students more easily use TI-Nspire to illustrate mathematical objects and to examine them thoroughly. Six teachers said that it gives more opportunities within problem-solving and that the students can manage more difficult tasks. But only three teachers definitely claim that the students seem to build a *deeper understanding* for mathematics with TI-Nspire. A reason for this that the teachers indicated is that deeper understanding always involves the use of paper and pencil. They believe that you can calculate and explore with the technology, but you need to transfer the results outside of it to really understand.

In this study, the three different technical combinations have appeared as the platform for TI-Nspire technology. Students and teachers have used this in regular education for a whole semester and have during this fairly long period of time been able to utilize almost all aspects of it that Artigue and Bardini [2] mention. The results from this study largely confirm their observations of the difficulties and the great efforts that meet students and teachers when they start using this technology. They also mention the substantial individual differences in how the instrumental genesis progresses. Some individuals benefit quickly from the technology, others will take a very long time. This is also described by Weigand and Bichler [13], and the results of this study show good compliance with their observations. Unfortunately, it was not possible in this study to give answers to all their questions, even if some light has been shed on some of them. For example, the findings suggest that there is a correlation between the uncertainty of dealing with the technical part of the unit and lack of knowledge about how to use it for the present problem. However, it seems that such deficiencies quickly can be removed for many students, if opportunities for collaboration in the classroom are given, and if the teacher encourages students to support each other in using the technology.

Teachers and students in the study showed significant progress in the instrumental genesis and also to some extent in the documental one. But here a much more complicated process is required, and the results suggest that this may take a long time, maybe several years. It is difficult to implement technology as an organic part of the resources of a "document" [6] which represent whole work sessions or lessons in mathematics. However, even here a certain development was observed, and there were signs of a continuation of the process involving the TI-Nspire for both teachers and students, now at a higher level.

The teachers were finally in the questionnaire asked if they thought that participation in this research project had been developing for them in their teaching. Five of them answered that it was to some parts, and three that it was in many ways. Some of their comments were:

T1: New teaching ways have been opened and I am interested to continue working with the software.

T2: It has developed me a lot, especially within data and statistics.

T3: Has been fun to see what is possible to do with the new technology. When my own knowledge and practice is better, I will benefit greatly from having seen all the possibilities.

6. Further research

In the research project, which was of a broad and in some respects superficial character, appeared some important questions that continued research could provide answers to. One such is based on a deeper analysis of the tns-files that the students produced in the problem-solving experiment. How did they argue in and for their solutions, how did they present them in text form, how could they take advantage of the dynamic links between different applications at the presentation?

Furthermore, it would be interesting to specifically monitor and analyse the documentational genesis, both of students and of teachers, over a longer period of time. The time span of the current study proved not to be enough. Finally, it would be of great interest to study how teachers can take advantage of the special capabilities of the platforms like TI-Nspire to assess students some of the abilities specified in the new Swedish curriculum, like ability to communicate and ability to present an argumentation and to give proofs.

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